Schedule Metric Algebraic Geometry MaSRW

Vincenzo Galgano Carlotta Langer Maximilian Wiesmann

L-VG: lecture by Vincenzo Galgano, **L-CL**: lecture by Carlotta Langer, **L-MW**: lecture by Maximilian Wiesmann, **SP**: student presentation

Logistics. The course takes place Tuesdays and Fridays from 11:10–12:40. Tuesdays we are in room WIL/A124, on Fridays we are in WIL/C103. (During the first 3 weeks it was Thursday instead of Tuesday.) First installment is Oct 15, last is Feb 05. There's a Christmas break with no courses between Dec 21 and Jan 03.

Seminar talks. Please tell us at least three weeks before the talk which talk you want to give—the earlier the better! Talks are first come, first serve. Presentations should be 60–75mins, with an additional 15mins of questions. Blackboard is fine.

Write-up. In addition to your seminar talk, you need to hand in a write-up of any of the other talks. Again, please let us know which one you would like to write. Deadline for the write-ups is February 12. Aim for around 5 pages written in LaTeX.

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Oct 15: Tensors I: tensors, tensor rank and varieties of rank-1 tensors (L-VG)

Oct 16: Tensors II: border rank and secant varieties (L-VG)

Oct 22: Tensors III: Apolarity theory and catalecticant maps (L-VG)

Oct 23: Equations of secant varieties of rational normal curves (L-VG)

Oct 29: Euclidean distance degree (L-MW)

Oct 30: Polar degrees (L-MW)

Nov 3: Seminar: The Alexander–Hirschowitz theorem (SP-VG) The student presents the proof of the theorem. For the most involved cases, the student can avoid formal details and prioritise the main ideas. Recommended references are [4], [16, §15].

Nov 6: Seminar: Algorithms for symmetric tensor decomposition (SP-VG) The presentation focuses on presenting the algorithms for rank decomposition of binary forms, namely the Sylvester algorithm and one between [1, 3].

Nov 10: Seminar: Identifiability and decompositions of tensors (SP-VG) The student can choose to present one of the following topics (☕ *denotes topics which require more work or more background*):

- Identifiability in secant varieties of Grassmannians [12, Sections 4 and 5];
- Advanced methods for Waring decomposition [20, either Section 3 or Section 4 (☕)];
- Waring loci and Strassen's Conjecture [6, choose three subsections among 3.1-3.5, and then Section 4];

Nov 13 Seminar:: Euclidean distance degrees for degenerate metrics (SP-MW) You should present the main theorems (Theorems 3.4 and 3.7) of [19]. Explain the full proof of Theorem 3.4, and the main ideas of the proof of Theorem 3.7. It's not necessary to present any of the machine learning motivation (though it's good for you to keep it in mind and connect with the lectures on Dec 17 & 18). Do your own "case study" where you compute the complex critical point set for a variety and a degenerate quadratic form of your choice (the example should be nontrivial...).

Nov 17: Attend Lectures at the Dresden-Leipzig day

Nov 20: Maximum likelihood estimation I: discrete and gaussian case (L-CL)

Nov 24: Maximum likelihood estimation II: maximum likelihood degree (L-CL)

Nov 27: Curvature of algebraic varieties (L-MW)

Dec 1: Volumes of tubular neighborhoods (L-MW)

Dec 4: Seminar: Iterative Proportional Fitting is an algorithm for calculating the MLE. (SP-CL) There are a wide range of sources available discussing this algorithm [9] [22, p. 7.3].

Dec 8: Seminar: Gauss–Bonnet via Weyl's tube formula (SP-MW) Present the proof of the Gauss–Bonnet theorem using Weyl's tube formula following Chapter 5 of [13]. Sections 5.2, 5.6 and 5.7 can be skipped entirely. As an example, compute both sides of the theorem independently for an algebraic hypersurface. **Jannis Kremer**

Dec 11: Seminar: Volumes of tubular neighborhoods of complete intersections (SP-MW) The goal is to explain Theorem 1.1 of [17] and its proof. Most of Sections 2 and 3 should already be known from the lecture (though you may need to adapt notation). Present Crofton's Theorem without proof. Section 4 should be explained in detail. Compute an example to illustrate Lemma 4.1.

Dec 15: Machine learning I: linear networks (L-CL)

Dec 18: Machine learning II: convolutional networks (L-CL)

Jan 5: Seminar: Geometry of Linear Neural Networks: Equivariance and Invariance Under Permutation Groups [15] (SP-CL) **Jonathan Karl Franz Kruske**

Jan 8: Seminar: On the expressive power of deep polynomial neural networks [14](SP-CL)

Jan 12: Machine learning III: learning varieties and sampling (L-CL)

Jan 15: Matrix multiplication tensor and complexity (L-VG)

Jan 19: Condition number (L-MW)

Jan 22: Seminar: Tropical Geometry of Deep Neural Networks [24] (SP-CL) Jannis Kremer

Jan 26: Seminar: Tensors in Metric Algebraic Geometry (SP-VG) Ilaria Pellegrini
The student can choose to present one of the following topics (☕ *denotes topics which require more work or more background*):

- Nonlinear Kalman varieties [21];
- Maximum likelihood estimation for tensor normal models via castling transforms [10];
- (☕) Maximum Likelihood degree via Schubert calculus [18, Section 3.B].
- (☕) Euclidean Distance degree of Veronese varieties [11, compare Proposition 7.10 and Corollary 8.7].

Jan 29: Seminar: On the complexity of matrix multiplication (SP-VG) The student can choose to present either [7], or one (or a combination) of [2] and [8, Proposition 6.2, Corollary 6.3].

Feb 2: Seminar: Probabilistic analysis of conic condition numbers (SP-MW) Introduce the concepts of *smoothed analysis* and *conic condition number*. Explain Theorems 20.1 and 21.1 of [5]. Do not give full proofs, instead focus on the basic ideas (e.g. Section 20.1) and on applications (Section 20.6 and 21.5).

Feb 5: Seminar: Computing volumes of semi-algebraic sets (SP-MW) Present the algorithm in [23] for high-precision numerical computation of convex semi-algebraic sets. Introduce the Weyl algebra, periods, holonomic functions and Picard–Fuchs operators carefully. Of course, you should run the algorithm and compute your own examples.

References

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